4.6 Notes

THEOREM 4.19 Log Rule for Integration

Let u be a differentiable function of x.

1. $\int \frac{1}{x}\,dx = \ln|x| + C$
2. $\int \frac{1}{u}\,du = \ln|u| + C$

Because $du = u'\,dx$, the second formula can also be written as

$$\int \frac{u'}{u}\,dx = \ln|u| + C.$$

Alternative form of Log Rule

EXAMPLE 1 Using the Log Rule for Integration

To find $\int 1/(2x)\,dx$, let $u = 2x$. Then $du = 2\,dx$.

Exploration

Integrating Rational Functions

Earlier in this chapter, you learned rules that allowed you to integrate *any* polynomial function. The Log Rule presented in this section goes a long way toward enabling you to integrate rational functions. For instance, each of the following functions can be integrated with the Log Rule.

$\dfrac{1}{2x}$	Example 1
$\dfrac{1}{4x-1}$	Example 2
$\dfrac{x}{x^2+1}$	Example 3
$\dfrac{3x^2+1}{x^3+x}$	Example 4(a)
$\dfrac{x+1}{x^2+2x}$	Example 4(c)
$\dfrac{1}{3x+2}$	Example 4(d)
$\dfrac{x^2+x+1}{x^2+1}$	Example 5
$\dfrac{2x}{(x+1)^2}$	Example 6

There are still some rational functions that cannot be integrated using the Log Rule. Give examples of these functions, and explain your reasoning.

EXAMPLE 2 Using the Log Rule with a Change of Variables

To find $\int 1/(4x - 1)\,dx$, let $u = 4x - 1$. Then $du = 4\,dx$.

Now you try... P. 351 #5

5. $\displaystyle\int \frac{2}{3x + 5}\,dx$

Now you try... P. 351 #21

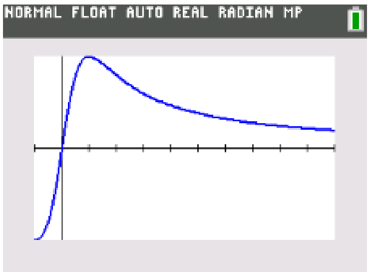
21. $\displaystyle\int \frac{(\ln x)^2}{x}\,dx$

EXAMPLE 3 Finding Area with the Log Rule

Find the area of the region bounded by the graph of

$$y = \frac{x}{x^2 + 1}$$

the x-axis, and the line $x = 3$.



4.6 Continued...

EXAMPLE 5 Using Long Division Before Integrating

See LarsonCalculusforAP.com for an interactive version of this type of example.

Find $\int \frac{x^2 + x + 1}{x^2 + 1}\,dx.$

Sometimes it's not so obvious . . .

Ex 3: $\int \frac{2x}{(x+1)^2}\,dx$

Now you try... P. 351 #19

19. $\displaystyle\int \frac{x^4 + x - 4}{x^2 + 2}\, dx$

EXAMPLE 4 Recognizing Quotient Forms of the Log Rule

a. $\displaystyle\int \frac{3x^2 + 1}{x^3 + x}\, dx = \ln|x^3 + x| + C$

b. $\displaystyle\int \frac{\sec^2 x}{\tan x}\, dx = \ln|\tan x| + C$

c. $\displaystyle\int \frac{x + 1}{x^2 + 2x}\, dx = \frac{1}{2}\int \frac{2x + 2}{x^2 + 2x}\, dx$

$\displaystyle= \frac{1}{2}\ln|x^2 + 2x| + C$

d. $\displaystyle\int \frac{1}{3x + 2}\, dx = \frac{1}{3}\int \frac{3}{3x + 2}\, dx$

$\displaystyle= \frac{1}{3}\ln|3x + 2| + C$

EXAMPLE 6 Change of Variables with the Log Rule

Find $\int \frac{2x}{(x+1)^2}\,dx$.

Guidelines for Integration

1. Learn a basic list of integration formulas. (By the end of Section 4.7, you will have 20 basic rules.)
2. Find an integration formula that resembles all or part of the integrand, and, by trial and error, find a choice of u that will make the integrand conform to the formula.
3. When you cannot find a u-substitution that works, try altering the integrand. You might try a trigonometric identity, multiplication and division by the same quantity, addition and subtraction of the same quantity, or long division. Be creative.
4. If you have access to computer software that will find antiderivatives symbolically, use it.

EXAMPLE 7 *u*-Substitution and the Log Rule

Solve the differential equation

$$\frac{dy}{dx} = \frac{1}{x \ln x}.$$

EXAMPLE 8 Using a Trigonometric Identity

Find $\int \tan x \, dx$.

EXAMPLE 11 Finding an Average Value

Find the average value of $f(x) = \tan x$ on the interval $[0, \pi/4]$.

EXAMPLE 9 Derivation of the Secant Formula

Find $\int \sec x \, dx$.

Integrals of the Six Basic Trigonometric Functions

$$\int \sin u \, du = -\cos u + C \qquad \int \cos u \, du = \sin u + C$$

$$\int \tan u \, du = -\ln|\cos u| + C \qquad \int \cot u \, du = \ln|\sin u| + C$$

$$\int \sec u \, du = \ln|\sec u + \tan u| + C \qquad \int \csc u \, du = -\ln|\csc u + \cot u| + C$$

EXAMPLE 10 Integrating Trigonometric Functions

Evaluate $\int_0^{\pi/4} \sqrt{1 + \tan^2 x}\, dx$.